# All Roads Lead to General Relativity

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# Alternative Roads to General Relativity

*Before 1915*: What were the conceptual resources of classical physics for a new theory of gravity?

- Several alternatives explored by Einstein's contemporaries (Renn et al. 2007)
- Einstein's distinctive approach

## Before 1915

### Field Theories of Gravity

- Nordström: scalar theory
- Mie, Hilbert: gravity and electromagnetism

### Astronomy

- Paradoxes of Newtonian cosmology (Seeliger)
- Empirical anomalies

### "Heretical Mechanics"

- Machian critique of Newton
- Geometrical formulation of Newton (Stachel's Newstein)

## Before 1915

Field Theories

Astronomy

"Heretical Mechanics"

## After 1915

Field Theories

- Dynamics for a spin-2 field

Astronomy

- Effective Field Theory for Newton
- Precision tests at solar system scales

Thermodynamics

- Black Hole Thermodynamics (Jacobson, Verlinde)

Induced Gravity

- GR "emerges" at one loop in QFT (Sakharov)

# Alternative Roads to General Relativity

*After 1915*: How does GR fit into the theoretical landscape?

- Alternative "derivations" of the field equations: what is assumed, and what is derived?
- How should we treat relationships among different theories?
- Robustness: multiple derivations as reply to historical contingency

# Outline

1. Newtonian Gravity as an Effective Field Theory
   - Methodological contrast
2. GR and Field Theory
   - Consistent theory of spin two fields
   - General constraints on classical field theories
3. Implications

# Fable of Wilton? (= Wilson + Newton)

*It is my contention that the concepts of Effective Theories, if understood and held by the early Newtonian scientists, would have led to a prediction that there must necessarily be an anomalous perihelion precession of Mercury and other planets, and that even the order of magnitude could have been guessed well before Le Verrier's announcement in 1859. There was no barrier to adopting these ideas in Newton's day, as it requires no new special experimental knowledge, nor knowledge of Einstein's relativity, but rather a more mature approach to how we think about the laws of nature. (Wells 2011, p. 14)*

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## Effective Field Theory

The theory should include all possible interactions compatible with fundamental symmetries.

# EFT of Newtonian Gravity (Wells 2011)

## Modifying Newton

Add terms to the Newtonian potential, compatible with Galilean symmetry:

$$V(r) = \frac{GMm}{r}\left[1 + \sum_{n=0}^{\infty} \mu_n \left(\frac{L}{r}\right)^n\right] \tag{1}$$

where $L$ is a length scale, and $\mu_n$ are dimensionless coefficients. (*Not* the most general form of interaction.)

## Observational Consequences

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- Modifications to $V(r)$
  - Introduce scales $\mu_i L$ for corrections to appear
  - Wells (2011): given "natural" scale set by speed of light $c$, plausible estimate for anomalous perihelion motion

# Implications

- Methodological Contrast
  - Clairaut, Newcomb, ...: proposed modifications of $V(r)$ criticized as *ad hoc*
  - "More mature approach": *expectation* of corrections, potential for astronomical evidence to set appropriate scales (or, more ambitiously, "predictions from naturalness")

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- Further Questions
  - First steps on a road to GR?
    *(Not clear how to proceed beyond fixing new scales.)*
  - Reformulate in Geometrized Newtonian theory?
  - EFT approach to Quanutm Gravity

# GR as Dynamics of a Spin-Two Field

## Feynman's "Venutians" (1962 lectures)

- Imagine successful particle physicists on Venus discover a new, very weak, attractive force
- Field theory treatment: simply add one more field to the collection, need to find consistent dynamics

## Flat space derivation of GR

- Massless spin-2 field, coupled to matter or self-coupled $\rightarrow$ unique consistent dynamics: Einstein's field equations

  (Feynman 1995 [1962]; cf. Gupta 1954; Kraichnan 1955,1956; Deser 1970)

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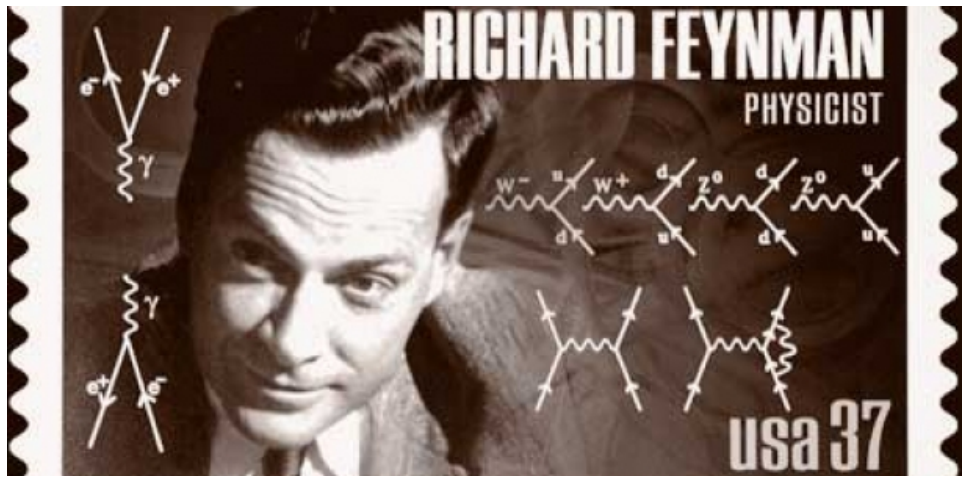
  - New dynamical variable $g_{ab}$ (defined in terms of $h_{ij}, \eta_{kl}$), such that $S$ is independent of initial flat spacetime metric
  - Non-linearities needed for consistent theory are precisely those of EFE
- Spacetime geometry determined by $g_{ab}$

*Einstein himself, of course, arrived at the same Lagrangian but without the help of a developed field theory, and I must admit that I have no idea of how he ever guessed at the final result. We have had troubles enough in arriving at the theory — but I feel as though he had done it while swimming underwater, blindfolded, and with his hands tied behind his back! ... (Feynman 1995 [1962], p. 87)*



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  - General motivations for these assumptions?

# What is a Classical Field Theory? (Darrigol 2014)

Given operational contrast between "test particles" and local fields, require that field dynamics should only depend upon features of the field measurable by particles (Faraday).

- P1 The action for the field theory includes a matter-dependent part and a field term which have the same symmetries.
- P2 For given velocities, the accelerations of two different test particles are proportional.

Argues that P1, P2 constrain possible field theories to Maxwell, Nordstöm, and Einstein.

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- What is derived?
  - In what sense have we recovered GR?
  - Not full space of solutions, but natural further steps for Venutian theorists
- Relation of GR to other theories
  - String theory: includes a massless spin-two exictation, therefore includes GR

# Robustness

- Advantages of Multiple Pathways
  - Value of independent lines of argument
  - Support for inevitability of GR (or something close to it)

# Pluralism

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  - Collection of basic principles, derived theorems
  - Different routes to GR: different choices regarding fundamental vs. derived consequence

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- Reflections on the Structure of Theories
  - Collection of basic principles, derived theorems
  - Different routes to GR: different choices regarding fundamental vs. derived consequence
- Modest pluralism
  - Against elevation of one set of principles as "constitutive," basic first principles

# Thank You